2021

Full Marks - 60

Time - 3 hours

The figures in the right-hand margin indicate marks

Answer *all* questions

Part-I

1. Answer the following : 1×8

   a) If $f(x) = |x - 1|$ then it is ____ at $x = 2$.

   b) Write the integrating factor of $\frac{1}{2}\frac{dy}{dx} + y \tan x = \log 2x$.

   c) $\vec{a} \times (\vec{b} \times \vec{c}) = (\vec{a}.\vec{c})\vec{b} -$ ____

   d) Write the condition for vector $\vec{A}$ and $\vec{B}$ are to be perpendicular.

   e) If $\vec{\nabla} \times \vec{A} = 0$, then $\vec{A}$ is a ____.

   f) $\delta(ax) = \frac{1}{|a|}$ ____.

   g) What is the value of $\vec{\nabla}.\vec{r}$?

   h) Write the mathematical form of Green's theorem in plane.

Part-II

2. Answer any ***eight*** of the following : $1\frac{1}{2} \times 8$

a) Prove that the position vector of a point is not selenodial.

b) Check if $f(x) = |x - 2|$ is continuous at $x = 2$.

c) Find particular integral of $\frac{d^2y}{dx^2} + y = \cos x$.

d) Solve $(2x + y)dy - (x + 2y)\,dx = 0$

e) Prove that $\left(\vec{a}.\vec{b}\right)^2 + \left(\vec{a} \times \vec{b}\right)^2 = a^2b^2$.

f) For what value of x the vector $\vec{A} = x\hat{i} + 2\hat{j} + 3\hat{k}$ and $\vec{B} = 2\hat{i} + x\hat{j} - 4\hat{k}$ are orthogonal.

g) Find grade ϕ if $\phi = \sqrt{x^2 + y^2 + z^2}$.

h) Prove that $\delta(-x) = \delta(x)$.

i) If $x = r\cos\theta$, $y = r\sin\theta$

Find $\frac{\partial(x, y, z)}{\partial(r, \theta, z)}\beta$.

j) Prove that $\vec{\nabla}.(\vec{u}v) = \vec{u}.\vec{\nabla}v + v(\vec{\nabla}.\vec{u})$.

Part-III

3. Answer any ***eight*** of the following : 2×8

a) Prove that $(2xy + 3)\,dx + (x^2 - 4y)dy = 0$ is an exact differential equation.

b) Find the approximate value of 9.9967.

c) State the condition of differentiability of $f(x)$ at $x = x_0$.

d) Find the value of $\lim\limits_{\substack{x\to 1\\ y\to 2}} \dfrac{2x^2y}{x^2+y^2+1}$.

e) Show that $\hat{i}\times(\vec{a}\times\hat{i}) + \hat{j}\times(\vec{a}\times\hat{j}) + \hat{k}\times(\vec{a}\times\hat{k}) = 2\vec{a}$

f) If $\vec{A}+\vec{B}+\vec{C}=0$ prove that

$$\vec{A}\times\vec{B} = \vec{B}\times\vec{C} = \vec{C}\times\vec{A}$$

g) Express the value of $\vec{\nabla}u$, $\vec{\nabla}v$ and $\vec{\nabla}w$ in terms of h_1, h_2, h_3 and unit vector $\hat{e}_1, \hat{e}_2, \hat{e}_3$.

h) Solve $xdx + ydy + 2(x^2+y^2)dx = 0$.

i) Evaluate $\int_{-1}^{+1} 4x^2\delta(2x+1)dx$.

j) Prove that

$$\text{div}(\vec{a}.\vec{b}) = \vec{b}.\vec{\nabla}\times\vec{a} - \vec{a}.\vec{\nabla}\times\vec{b}.$$

Part-IV

4. a) Solve the differential equation
$(D-2)(D+1)^2y = e^{2x} + e^x$. 6

OR

b) If $x^y + y^x = (x+y)^{x+y}$ find $\frac{dy}{dx}$ at $x = 1, y = 1$.

5. a) State and prove the Euler's Theorem. 6

OR

b) Define vector triple product. Prove that
$\vec{A} \times (\vec{B} \times \vec{C}) = \vec{B}(\vec{A}.\vec{C}) - \vec{C}(\vec{A}.\vec{B})$

6. a) Find expression for Curl in term of orthogonal curvilinear coordinates. 6

OR

b) Define Dirac delta function
Prove that $\delta(x) = \lim_{k\to\infty} \frac{\sin^2 kx}{\pi k x^2}$.

7. a) State and prove that Gauss divergence theorem. 6

OR

b) i) If $\vec{a}$ is a constant vector,
Prove that grad $(\vec{r}.\vec{a}) = \vec{a}$. 3

ii) Find Curl of $\vec{F}$ if $\vec{F} = \frac{x\hat{i} + y\hat{j}}{x+y}$. 3

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Answer *all* questions

Part-I

1. Answer the following : 1×8

a) State the direction of Torque.

b) Define Non inertial frame.

c) State Hooke's law of elasticity.

d) Write unit and dimension of co-efficient of Viscosity.

e) Kepler's 3rd law in known as _____ law.

f) GPS statellites carry atomic clocks with them. (True/false)

g) Draw graph of energy verses displacement for body in SHM.

h) Write the relativistic energy momentum relation.

Part-II

2. Answer any ***eight*** of the following : $1\frac{1}{2} \times 8$

a) Prove law of conservation of angular momentum of particle.

b) Define a flywheel.

c) Explain laboratory frame of reference.

d) Find the torque of force $\vec{F} = \left(-\hat{i} - 3\hat{j} + \hat{k}\right)$ N about a point (1m, 1m,1m) when the force act at point P(2m, 3m, 4m).

e) Prove that Workdone in elongation a wire is $\frac{1}{2} \times$ load $\times$ elongation.

f) Calculate the horizontal force required to move plate of area 150cm^2 kept on the upper surface of viscous liquid of thickness 3mm flowing at a speed 5cm/s.

g) If central force is $\vec{F} = \frac{-k\vec{r}}{r^3}$. Find its potential energy.

h) How an astronaut feels weightless in a satellite.

i) Explain critical damping.

j) Write equation for time dilation.

Part-III

3. Answer any ***eight*** of the following : 2×8

a) A body mass 15gm appears 20gm when in motion, find its velocity.

b) Find the length of Second's pendulum.

c) Explain Pseudo-force with examples.

d) Two bodies of masses 3kg and 5kg are located at point (1, 2) and (–1, 3) respectively. Calculate the coordinate of centre of mass.

e) Discuss the variation of viscocity of liquids and gases with rise in temperature.

f) Explain GPS.

g) Prove that Kepler's 2nd law from conservation of angular momentum of central force motion.

h) Derive relation between graviational potential and graviational field.

i) Derive torque is term of mement of intertia of a body.

j) State postulates of special theory of relativity.

Part-IV

4. a) State parallel axis theorem. Find expression for moment of inertia of a solid cylider about the axis passing through its own axis of symmetry. 6

OR

b) Explain non inertial frame of Reference and Fictious force. Find the expression for total force and fictitious force acting on a body in non-inertial frame of reference.

5. a) Derive an expression for depression produced in a single cantilever at it's free end. 6

OR

b) Derive the relation between elastic constant (γ, β, η).

6. a) What is central force ? Obtain equation of motion and first integral. 6

OR

b) Explain the reduced mass of two body problem. Its reduced mass is always smaller than either of masses.

7. a) Derive expression for total energy of a particle executing S.H.M. Find the distance from mean position where K.E. is $\frac{1}{2}$ potential energy. 6

OR

b) Derive Lorentz transformation equation.

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Answer ***all*** questions from any one section

SECTION-A

Part-I

1. Answer the following by fill in the blanks or one word answer : 1×8

   a) The Torque acting on a body of moment of inertia(I) and angular acceleration (α) are related by ___.

   b) Under what condition the law of conservation of angular momentum hold ?

   c) What is relation between Graviational potential and Gravitational field.

   d) Central force which acts on a particle such that it is directed towards or away from ___.

   e) The relaton between Yaung's modulus(γ), rigidity modulus (η) and Poisson's ration(σ) is ___ .

   f) Does Poiseuille's equation need correction ?

g) The differential equation of SHM is ____.

h) What is the formula for rest energy of a body.

Part-II

2. Answer any ***eight*** of the following : $1\frac{1}{2} \times 8$

a) Derive Torque due to many forces acting at a point.

b) Write two example of conservation of Angular momentum.

c) Give one application of coriolis force and explain.

d) Prove that the orbital speed of a satellite very close to earth is nearly 8km/s.

e) Find the reduced mass of Hydrogen atom.

f) Give three application of Artificial satellites.

g) What do you mean by angle shear and angle of twist.

h) A plate of area $200cm^2$ is placed on the upper surface of castor oil 4mm thick. Find the horizontal force necessary to move the plate with velocity 2m/s, η=15.5 poise.

i) Derive differential equation of SHM.

j) Find the frequency of a body in SHM if it takes 6 second from its mean position to extreme position.

Part-III

3. Answer any ***eight*** the following : 2×8

a) State and explain perpendicular axis Theorem.

b) Two body masses 2kg and 5kg are located (1, 2) and (1, –3) respectively. Calculate the coordinate of center of mass.

c) How gravitational potential at internal point of a spherical shell vary with its radius.

d) An annular disc of mass 0.1kg and radius 0.2m and 0.25m rolls such that centre has velocity 1m/s. Find kinetic energy.

e) A wire of length 2m and diameter 1mm is clamped at one end, find the couple needed to twist the other end by 90°, $\eta = 2.8 \times 10^{11}$ dyne/cm^2.

f) Explain critical damping.

g) Write relation between sharpness resonance and Q-factor.

h) Why it is easier to wash our dress by detergent add water.

i) Compare radius of two copper wire of same length loaded by two mass 2kg and 4kg producing elongated in the ratio 1 : 4.

j) Find the distance from mean position at which kinetic energy is half of its total energy of a particle executing SHM of amplitude 20cm.

Part-IV

4. a) Derive moment of inertia of solid cylinder about its own axis. 6

OR

b) State and prove law of conservation of Angular momentum of a system.

5. a) Derive gravitational potential at a point laying outside of spherical shell. 6

OR

b) Define central force. Obtain equation of motion and first integrals.

6. a) Derive relation between, the elastic constant (γ, β, η). 6

OR

b) Derive the expression for Poiseuille's formula.

7. a) Set up differential equation of motion for forced vibration. Find its solution. 6

OR

b) Explain Lissajous figures for superposition of two orthogonal simple harmonic vibration with 1 : 1 frequency.

SECTION-B

Part-I

1. Answer the following by fill in the blanks or one word answer : 1×8

a) Surface tension of liquid ___ with rise in temperature.

b) What is the value of moment of inertia of a body in term of radius of gyration.

c) There is a phase difference of ___ between velocity and acceleration of a particle SHM.

d) Time frictional force or damping force is directly proportional to the velocity of particle in damped oscillation. (True/False)

e) Entropy of a system in equllibrium is ___.

f) If the temperature of source is increased, the efficiency of carnot engine is ___.

g) Stready magnetic field are governed by ___ law.

h) What is value of ripple factors of half wave rectifier.

Part-II

2. Answer any ***eight*** of the following : $1\frac{1}{2} \times 8$

a) Derive moment of inertia of solid cylinder about a diameter of one of its face.

b) A wheel mounted on a shift of negligible moment of Inertia is rotating at 500rpm. An exactly similar second wheel initially at rest is suddenly coupled to the same shift. Find the angular velocity of combination.

c) Define Poisson's ratio. What is its unit ?

d) A particle of mass 1kg execute SHM subjected to potential energy $20x^2$J. Find the frequency of oscillation.

e) Derive Time period of a damped harmonic oscillation.

f) State Carnot's Theorem.

g) What is the importance of T-S diagram.

h) Calculate the number of electric lines of force originating from a charge of 10c. Given $\epsilon_0 = 8.854 \times 10^{-12} c^2 N^{-1} m^{-2}$.

i) What is the importance of LCR series A.C. circuit.

j) A potential difference of 100V is applied to a coil of L = 10H and R=100Ω. After how much time will be 0.5A ?

Part-III

3. Answer any ***eight*** of the following : 2×8

a) Discuss the relation between surface tension and surface energy.

b) Prove that $\sigma = \frac{3\beta - 2\eta}{6\beta + 2\eta}$.

c) Distinguish between Amplitude resonance and velocity resonance.

d) Show that $\frac{1}{\sqrt{\mu \in}}$ has dimension of velocity.

e) Discuss critical damping.

f) Why $C_p > C_v$.

g) Write Maxwell thermodynamic relation.

h) Calculate the entropy of 1kg of water heated from 30°C to 60°C. Given specific heat of water C = 1cal/gm/°C.

i) What is p-type semiconductor and it can be produced.

j) Distinguish between JFET and BJT.

Part-IV

4. a) Derive moment of inertia of solid sphere about diameter. 6

OR

b) Derive Poiseuille's formula for the rate of flow of liquid through a tube.

5. a) What is forced vibration ? Establish its differential equation of motion and solve it. 6

OR

b) Expression for composition of two SHM propagating perpendicularly to each other with 2 : 1 frequency.

6. a) Prove that the entropy of a thermodynamic system remains constant in any reversible process. 6

OR

b) Derive the differential equation for the flow of heat along bar during steady state and obtain its solution.

7. a) State Biot Savart's law. Derive magnetic induction due to a current carrying circular coil on the axis. 6

OR

b) Obtain an expression for the instantaneous current during growth and decay in an LR circuit containing d.c. source.